TRINITY COLLEGE

Year 11 Mathematics Specialist
Test 1 2016

Calculator Assumed
Combinatorics and Logic

STUDENT'S NAME ______________________________

DATE: **TIME:** 50 minutes **MARKS**: 54

INSTRUCTIONS:

Standard Items: Pens, pencils, ruler, eraser.

Special Items: Three calculators, drawing instruments, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (6 marks)

Consider the true statement:
"If a quadrilateral is a rectangle, then it has two pairs of parallel sides"

(a) Write down the converse of this statement and state whether it is true or false, if it is false, provide a counter-example. [2]

If a quadrilateral has 2 pairs of // sides, then it is a rectangle [FALSE] could be a square, or //ogram, ..

(b) Write down the contrapositive of this statement and state whether it is true or false, if it is false, provide a counter-example. [2]

If a quadrilateral does not have two pairs of parallel sides, then it is not a rectangle [TRUE]

(c) Write down the inverse of this statement and state whether it is true or false, if it is false, provide a counter-example. [2]

If a quadrilateral is not a rectangle, then it does not have two pairs of parallel sides [FALSE] square, //ogram, ..

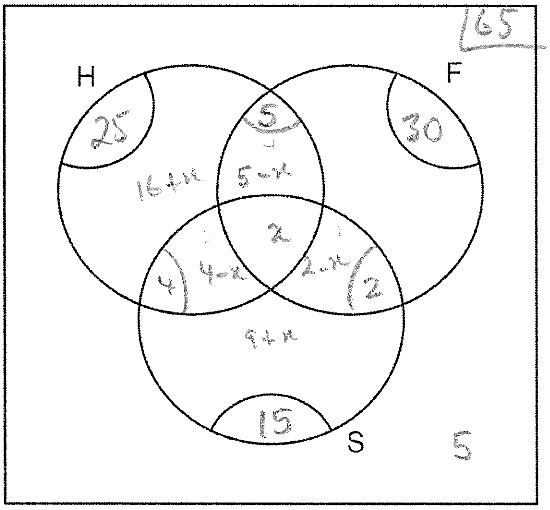
2. (2 marks)

A small primary school has 500 students. By using the Pigeon Hole Principal, show that at least two of them were born on the same day of the year.

There are 365 (366) days in a year, ∴ by PHP there must be at least two students born on the same day of the year

3. (6 marks)

In a group of 65 students, 30 played football, 25 played hockey, 15 played soccer, 5 played football and hockey, 4 played hockey and soccer, 2 played soccer and football and 5 played none of the three sports.
By using the Venn diagram below or otherwise, determine:



$30 + 16 + x + 4 - x + 9 + x + 5 = 65$

$59 + x = 60$

$x = 1$

(a) How many students play all three sports? [4]

1

(b) How many students play at least two sports? [1]

9 [4 + 3 + 1 (+ 1) all the]

(c) How many students play either football or hockey? [1]

50

4. (10 marks)

During the late 1980's motor vehicle number plates showed a single digit, followed by any two letters, followed by any 3 digits, as shown. (Assume digits and letters may be repeated)

8 AB 012

(a) How many different number plates begin with **8 A B**? [1]

$10.10.10 = \underline{1000}$.

(b) How many different number plates begin with **8** and end with **8 8 8**? [1]

$26.26 = \underline{676}$

(c) For number plates beginning with the digit **8**, how many different plates are possible? [1]

$1000 \times 676 = \underline{676\,000}$.

(d) 60 771 motor vehicles were newly registered in WA for the year ending 30 June 1989. Assume that new vehicles continue to be registered at this rate, and that different plates are issued for each vehicle. For how long, to the nearest month, could the series of plates beginning with **8** be expected to last? [3]

$\frac{676\,000}{60\,771} = 11.124 = 11\text{yrs } 1\text{month}$

(e) How many different plates of the series beginning with the digit **8** have the 2 letters the same and the 3 final digits the same? [2]

$26.1 \times 10.1.1 = 260$

The sample number plate shown above has two consecutive letters in alphabetical order, followed by three consecutive digits in ascending order.

(f) How many different plates commencing with **8** have two consecutive letters in alphabetical order, followed by three consecutive digits in ascending order? [2]

$25.1 \times 8.1.1 = 200.$

no 9 or 8

5. (7 marks)

A car has a seating capacity of five with only two allowed in the front. How many ways are there of seating five people if:

(a) only two can drive? [2]

$$\underset{\uparrow \atop \text{drive}}{2} . 4 . 3 . 2 = 48$$

(b) only one can drive and another refuses to sit in the back? [2]

$$\underbrace{1}_{\text{driver}} . \underbrace{1}_{\text{in front}} . \underbrace{3 . 2}_{\text{back}} = 6$$

(c) any of the five can drive but two refuse to sit together? [3]

sit together
front : $2 \times \underbrace{(3 \times 2 \times 1)}_{\text{back}}$
back : $\underline{2 . 1 . 3 \times 2} \times \underbrace{2}_{\text{front}}$

$= 36$ ∴ not together $= 5! - 36 = \underline{\underline{84}}$

6. (4 marks)

Show that $\binom{n}{n-2} = 10$ simplifies to $n(n-1) = 20$. Hence, solve for n.

$$\frac{n!}{(n-2)!\,2!} = 10 \Rightarrow \frac{n(n-1)\cancel{(n-2)!}}{2 . \cancel{(n-2)!}} = 10$$

$$\Rightarrow n(n-1) = 10$$

ie $n^2 - n - 10 = 0$

$(n+4)(n-5) = 0$ ∴ $\underline{n = 5}$ as n is non negative

7. (11 marks)

(a) Using the digits 0, 1, 2, 3, 4, 5, 6 without repetition

(i) How many 3 digit numbers are greater than 450? [2]

$1.1.3 + 1.1.4 + 2.5.4 = 47.$

$4\,5\,-$ $\quad 4\,6\,-$ $\quad \begin{matrix}5\\6\end{matrix}\,-\,-$

no zero

(ii) How many 3 digit numbers are divisible by 5? [2]

$\underline{5}\ \underline{4}\ \frac{1}{0} + \underline{4}\ \underline{4}\ \frac{1}{5} = 36$

(iii) How many numbers are less than 1000? [3]

1 digit : 6

2 digit : 5.5 = 25 $\quad$ = 131

3 digit : 5.5.4 = 100

can't start with zero

(b) In how many ways can 3 men be selected from 12 men:

(i) If one of the men is to be included in every selection? [1]

$\binom{1}{1}\binom{11}{2} = 55$

(ii) If two of the men are to be excluded from every selection? [1]

$\binom{10}{3} = 120$

(iii) If one man is to always be included and two men are to always be excluded? [2]

$\binom{1}{1}\binom{9}{2} = 36$

8. (5 marks)

There are ten chairs in a row.

(a) In how many ways can two people be seated? [2]

$\binom{10}{2} \times 2! = 90$

(b) In how many of these ways will the two people be sitting in adjacent chairs? [2]

$\binom{9}{1}\binom{1}{1} \times 2 = 18$

(c) In how many ways will they have at least one chair between them? [1]

$90 - 18 = 72$

9. (3 marks)

If $\binom{28}{r-4} = \binom{28}{r-2}$, determine the value of r. (show all working)

$28-(r-2)$

$\binom{n}{r} = \binom{n}{n-r}$

13 15

$\therefore r-4 = 30-r$

$2r = 34$

$\underline{r = 17}$

$28-(r-4) \qquad 28-(r-2)$

$$\frac{\cancel{28!}}{(32-r)!\,(r-4)!} = \frac{\cancel{28!}}{(30-r)!\,(r-2)!}$$

$$1 = \frac{(32-r)!\,(r-4)!}{(30-r)!\,(r-2)!}$$

$$= \frac{(32-r)(31-r)\cancel{(30-r)!}\,\cancel{(r-4)!}}{\cancel{(30-r)!}\,(r-2)(r-3)\cancel{(r-4)!}}$$

$$1 = \frac{(32-r)(31-r)}{(r-2)(r-3)}$$

$(r-2)(r-3) = (32-r)(31-r)$ Solve $r = 17$